CISC 1100: HW 2

SOLUTIONS

Let $U = \{1, 2, ..., 15\}$ and let:

$$A = \{x \in U | x \text{ is divisible by } 2\}$$

$$B = \{x \in U | x \text{ is divisible by } 3\}$$

$$C = \{x \in U | x \text{ is divisible by } 6\}$$

$$D = \{x \in U | x \text{ is divisible by } 4\}$$

1) a) Find $|A \cap D|$.
$|A \cap D| = |D| = 3$
b) Find $|A \cup D|$.
$|A \cup D| = |A| = 7$
c) Find $|D \cup B|$.
$|D \cup B| = |D| + |B| - |D \cap B| = 3 + 5 - 1 = 7$
d) Find $|A \cap B|$ and $|C|$.
$|A \cap B| = |C| = 2$

2) Suppose that in a poll of 100 people, 65 people read *Keep it sharp*, 50 people read *Who*?, and 80 people read either magazine.
a) How many people read both magazines?
$|K \cup W| = |K| + |W| - |K \cap W|$
$\therefore 80 = 65 + 50 - |K \cap W|$
$|K \cap W| = 35$
b) How many people read neither magazine?
Let U be the universal set; then $|U| = 100$ and $|(K \cup W)'| = |U| - |K \cup W| = 100 - 80 = 20$.
3) Consider the proposition $[(p \wedge q) \vee (q \wedge r)] \Rightarrow (p \wedge r') \vee q$.
a) As done in class and the notes, write a tree for this proposition starting with $\Rightarrow$.
Ask for a picture.
b) What is the truth value of this proposition when p, q are true and r is false?
True.
4) a) Write a truth table with the following columns: $p; q; q'; p \Rightarrow q; p \wedge q'$.
Ask for picture.
b) This table demonstrates a new method of proof: 'proof by contradiction.' Given a proposition of the form "If p, then q", we assume $p \wedge q'$ and show that this is false. Let's

work on an example. Consider the proposition

$$x > 3 \Rightarrow \frac{1}{x^2} + 1 < \frac{5}{4}$$

Name p, q and $p \wedge q'$.

$p := x > 3$

$q := \frac{1}{x^2} + 1 < \frac{5}{4}$

$p \wedge q' := x > 3 \wedge \frac{1}{x^2} + 1 \geq \frac{5}{4}$

c) Show that $p \wedge q'$ is false.

Let $x = 10$. Then $x = 10 > 3$ but $\frac{1}{x^2} + 1 = \frac{1}{100} + 1 = 1.01$, which is not greater than $5/4 = 1.25$.

d) Use your result (the proposition in (b)) to prove the following theorem:

If n is a prime number greater than 3, then $4n^2 + 4 < 5n^2$

The proposition holds for all real numbers, so it applies to any prime. Therefore if $n > 3$, then $\frac{1}{n^2} + 1 < \frac{5}{4}$. But getting a common denominator:

$$\frac{n^2+1}{n^2} < \frac{5}{4}$$

Cross multiplying, $4n^2 + 4 < 5n^2$