CISC 1100: HW 2

NAME:

Let $U = \{1, 2, ..., 15\}$ and let:

$$A = \{x \in U | x \text{ is divisible by } 2\}$$
$$B = \{x \in U | x \text{ is divisible by } 3\}$$
$$C = \{x \in U | x \text{ is divisible by } 6\}$$
$$D = \{x \in U | x \text{ is divisible by } 4\}$$

1) a) Find $|A \cap D|$.

b) Find $|A \cup D|$.

c) Find $|D \cup B|$.

d) Find $|A \cap B|$ and $|C|$.

2) Suppose that in a poll of 100 people, 65 people read *Keep it sharp*, 50 people read *Who?*, and 80 people read either magazine.

a) How many people read both magazines?

b) How many people read neither magazine?

3) Consider the proposition $[(p \wedge q) \vee (q \wedge r)] \Rightarrow (p \wedge r') \vee q$.
a) As done in class and the notes, write a tree for this proposition starting with $\Rightarrow$.

b) What is the truth value of this proposition when p, q are true and r is false?

4) a) Write a truth table with the following columns: $p; q; q'; p \Rightarrow q; p \wedge q'$.

b) This table demonstrates a new method of proof: 'proof by contradiction.' Given a proposition of the form "If p, then q", we assume $p \wedge q'$ and show that this is false. Let's work on an example. Consider the proposition

$$x > 3 \Rightarrow \frac{1}{x^2} + 1 < \frac{5}{4}$$

Name p, q and $p \wedge q'$.

c) Show that $p \wedge q'$ is false.

d) Use your result (the proposition in (b)) to prove the following theorem:

If n is a prime number greater than 3, then $4n^2 + 4 < 5n^2$