

CISC 1100: HW 3

SOLUTIONS

1) Let the predicate " $apple(s, d)$ " mean that "Tammy will make an apple pie if at least 4 people demand it and she has a supply of at least 6 apples."

a) Why is this a predicate and not a proposition?

Its truth value depends on two variables, s and d .

b) State the variables of the predicate in 'plain English'.

s := Supply of apples.

d := Demand of pies.

c) Give values for s, d showing that the following quantifiers are true: $\forall s \exists d : apple'(s, d)$.

$d = 3$ works. If 3 people demand pies, Tammy will not make the pies because there is too little demand, regardless of how many apples she has.

d) Give values for s, d showing that the following quantifiers are true: $\exists s \exists d, apple(s, d)$.

$s = 6, d = 4$ works. So will any larger value for either variable.

e) Suppose that $S = \{(s, d) | s > 5, d > 5\}$. Is it true that $\forall (s, d) \in S, apple(s, d)$? Why?

This is true. Since $s > 5$, s is at least 6 as required. Since $d > 5$, d is at least 6, which is greater than the required 4.

2) Consider the following ancient math proposition:

Let a number be given. Square the number and make it less by one. On the other hand, make the number less by one and make the number more by one, and then multiply these two. These products are the same.

a) Suppose that the "number ... given" is x . Rewrite this as a proposition depending on x . (Hint: an equation is a proposition.)

$$x^2 - 1 = (x + 1)(x - 1)$$

b) What values of x , if any, make this proposition false? Why?

This proposition is true for all real numbers (and all complex numbers, if you're curious).

3) Consider the following propositions:

Any albatross can fish.

A man who knows how to fish will eat for the rest of his life.

Any man is an albatross if he brings bad news.

a) Name appropriate propositional variables and restate these statements in logic notation.

a := is an albatross.

m := is a man.

f := knows how to fish.

b := brings bad news.

e := eat for the rest of his life.

b) Show that if a man brings bad news, he will eat for the rest of his life.

$m \wedge b \Rightarrow a$

$a \Rightarrow f$

$\therefore m \wedge b \Rightarrow f$

$f \Rightarrow e$

$\therefore m \wedge b \Rightarrow e$

4) Let $A = \{1, 2, 5, 6, 8\}$ and define a relation r on A by the rule $x - y$ is even.

a) Write r in set builder notation.

$r = \{(x, y) \in A \times A \mid x - y \text{ is even}\}$

b) Write r in set lister notation. (Write out all the elements in the relation.)

$r = \{(1, 5), (5, 1), (2, 6), (6, 2), (2, 8), (8, 2), (6, 8), (8, 6), (1, 1), (5, 5), (2, 2), (6, 6), (8, 8)\}$