CISC 1100: HW 4

NAME:

1) a) Write the set of all numbers $x \in \mathbb{Z}$ such that $x \equiv 4(mod\,5)$ in set builder notation.

b) State the equivalence classes for the relation "$x \equiv y(mod\,5)$" on $S = \mathbb{Z}$

c) Find all solutions to $2x \equiv 3(mod\,5)$. Hint: you only need to test one element from each equivalence class.

d) Find all solutions to $5x \equiv 4(mod\,6)$ and $2x \equiv 4(mod\,7)$. (Notice you have different equivalence classes now.)

e) Generally we are looking at equations of the form $ax \equiv b(mod\,n)$. What pattern do you notice is emerging between a, n? This condition is necessary to guarantee a solution; for example $2x \equiv 1(mod\,4)$ has no solution.

2) Modular arithmetic has many applications. One is determining powers of $i = \sqrt{-1}$, the imaginary number.

a) Write out the powers of $i^0, i^1, i^2, ..., i^7$.

b) Notice that the powers of i 'reset' every 4; for instance $i = i^5$. Therefore $i^k = i^0, i^1, ..., i^3$ depending on what k is equal to $(mod\,4)$. Using this fact, find i^{107}, i^{-22}

c) Another application is the 'last digit' problem. Find $12(mod\,10), -28(mod\,10), 217(mod\,10)$

d) What pattern do you notice emerging?

e) If $a \equiv a'(mod\,n)$ and $b \equiv b'(mod\,n)$, then $ab \equiv a'b'(mod\,n)$. Using this fact and your observation in (d), find the last digit of $(22)^{17}$.

3) Suppose that X is a set and let $S = P(X)$, the power set. Consider the relation $r = \{(A, B) \in S \times S | A \subset B\}$.

a) Prove that r is irreflexive.

b) Prove that r is antisymmetric.

c) Prove that r is transitive.

d) Why isn't it necessary to show that r is reflexive or symmetric?

As we learned in class, these properties make r an 'inequality relation.' An important aspect of inequality relations is that, for any finite set S with inequality relation r, S contains a 'least element,' that is, $\exists x \in S, xRy\ \forall y \in S$. (Think of this as $0 < n$ for any natural number$n \in \mathbb{N}$).

e) Assume X is finite. What is the least element for this relation r on $S = P(X)$?

Another important aspect of the relation $<$ is called the 'well-ordering principle.' This is: $\forall x, y \in S, x \neq y, xRy$ or yRx. (E.g., given any two distinct numbers, one is always larger than the other.)

f) Is the relation r on $S = P(X)$ well ordered? Why or why not?

4) Consider the relation: $S = \{a, b, c\}; r = \{(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, c)\}$

a) Write the diagram for this relation (with nodes at elements of S and arrows representing 'is related to').

b) By use of your diagram, classify this relation (reflexive/irreflexive/neither, symmetric/anti... etc.).