CISC 1100 - HW 6

NAME:

1) Prove the following formula via induction:

$$\sum_{n=0}^{k} a_0 r^n = a_0(\frac{1-r^{k+1}}{1-r})$$

a) State predicate $p(k)$ which we are proving.

$$p(k) : \sum_{n=0}^{k} a_0 r^n = a_0(\frac{1-r^{k+1}}{1-r})$$

b) Show the base case when $k = 0$. This may feel rather obvious.

$$\sum_{n=0}^{0} a_0 r^n = a_0$$

$$a_0(\frac{1-r^{0+1}}{1-r}) = a_0$$

c) Complete the proof by showing that $p(k) \Rightarrow p(k+1)$.
We assume:

$$\sum_{n=0}^{k} a_0 r^n = a_0(\frac{1-r^{k+1}}{1-r})$$

We add $a_0 r^{k+1}$ to the right hand side. (Doing so on the LHS is obvious.)

$$a_0(\frac{1-r^{k+1}}{1-r}) + a_0 r^{k+1} = a_0(\frac{1-r^{k+1}}{1-r} + \frac{r^{k+1}-r^{k+2}}{1-r}) = a_0(\frac{1-r^{k+2}}{1-r})$$

2) Write out the full Euclidean algorithm to find the GCD of 36, 42.

$$36 = 42(1) - 6$$

$$42 = 6(7) - 0$$

So, the GCD is 6.

3) a) In five years, what is the future value of an account with 4.2% annual interest rate, compounding quarterly (4 times a year), if \$5,000 is invested today?

$$5000(1 + \frac{.042}{4})^{(5)(4)} \approx \$6161.64$$

b) Suppose that you have another option, with 3.9% annual interest rate but compounding monthly. Calculate the future value of this account with the same $5,000 investment. Which is the better option?

$$5000(1 + \frac{.039}{12})^{(5)(12)} \approx \$6074.63$$