

CISC 1100 - HW 7

NAME:

1) Let's prove the future value formula by induction. The formula is (using i, n):

$$A = P(1 + i)^n$$

a) State predicate $p(n)$ which we are proving. Hint: this is the formula.

$$p(n) : A = P(1 + i)^n$$

b) Show the base case when $n = 0$. Hint: plug in 0 for n .

$p(0) : A = P(1 + i)^0 = P$. This is true since the future value at 'period 0' is just the initial principle, P .

c) Complete the proof by showing that $p(n) \Rightarrow p(n + 1)$. Hint: $p(n)$ is the future value on period n , and $p(n + 1)$ is the future value at period $n + 1$. Plug in A from $p(n)$ as P for $p(n + 1)$ and compute.

Assume that $A_n = P(1 + i)^n$ is true, where A_n is the future value in period n . Then since $A_{n+1} = A_n + A_n i = A_n(1 + i)$, and $A_n = P(1 + i)^n$, we must have that $A_{n+1} = P(1 + i)^n(1 + i) = P(1 + i)^{n+1}$.

This is not the only way to do this.

2) You are offered 3 investment opportunities: a savings account at 3.4% compounding daily, a savings account at 3.8% compounding monthly, or an annuity at 4.8% compounding monthly. If you are investing \$5000 over a 3 year period, compute the future value of each option and determine which is best.

$$3.4\%: A = 5000(1 + .034/365)^{(3)(365)} \approx 5536.89$$

$$3.8\%: A = 5000(1 + .038/12)^{(3)(12)} \approx 5602.75$$

$$4.8\%: A = \frac{5000}{(3)(12)} \left(\frac{(1 + .048/12)^{(3)(12)} - 1}{.048/12} \right) \approx 5366.40$$

The best option is the 3.8% account.

3) a) How many ways can a 5 person committee be chosen from 17 students?

No replacement, order doesn't matter; combination. ${}_{17}C_5 = \frac{17!}{5!12!} = 6188$.

b) There have been 45 presidents. If I make a second Mount Rushmore, how many ways can I do so? (There are 4 presidents on Mount Rushmore.)

No replacement, order matters; permutation. ${}_{45}P_4 = \frac{45!}{41!} = 3,575,880$.

4) a) If we are electing a president, vice president and secretary from our 22 person class, what is the probability you are president?

Sample space: Ways to elect president, vice and secretary. No replacement, order matters; permutation. ${}_{22}P_3 = 9240$.

Event: You are elected president. Therefore $\underline{1} \cdot \underline{21} \cdot \underline{20} = 420$. (You are president, 21 remaining for vice, 20 remaining for secretary).

Thus the probability is $\frac{420}{9240} = \frac{1}{22} \approx 4.5\%$.

b) What's the probability that you are vice president?

This is exactly the same probability; counting the event yields $\underline{21} \cdot \underline{1} \cdot \underline{20} = 420$. (Not you for president, you for vice, not you or the president for secretary.)

Thus the probability is $\frac{420}{9240} = \frac{1}{22} \approx 4.5\%$.

c) What's the probability that you get any position?

You are president, or you are vice president, or you are secretary. Becoming secretary has the same probability as our answers in (a), (b).

Thus the probability is $\frac{420}{9240} + \frac{420}{9240} + \frac{420}{9240} = \frac{3}{22} \approx 13.6\%$.